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Stefan Banach

by

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Stefan Banach was born on 20 March 1892, in Cracow. His father, an official in the railway administration, Greczek by name, came of a peasant family living in the highlands. The particulars of Banach's early childhood are unknown; however, we know that immediately after his birth he was put under the guardianship of a laundress, living in a garret in Grodzka Street (nr 70 or 71); Banach was the name of her husband. Since that time Banach never saw his mother, so that practically he did not know her. Neither did his father care for him. Since the age of 15, Banach had to support himself by private teaching. He was very keen on giving coaching lessons in mathematics. As far as mathematics goes, he was selftaught. We do not know how or when he acquired the knowledge of French but we do know that as a schoolboy he read Tannery's "Introduction à la théorie des fonctions". Before the First World War, he attended lectures delivered at Jagellon University by S. Zaremba, but he did so irregularly and only for a short time. A little later, he moved to Lwów Institute of Technology, and there he passed what was called the "first examination" certifying that he had studied engineering for two years. When, in 1914, the First World War broke out, Banach returned to Cracow. One summer evening, in 1916, as I was walking along the "Planty" (1), I heard a conversation, or rather only a few words. I was so struck by the words "the Lebesgue integral" that I came nearer to the bench on which the speakers were sitting and, then and there, I made their acquaintance. The speakers, Stefan Banach and Otto Nikodym, were discussing mathematics. They told me they had another chum — Wilkosz. It was not only mathematics that bound together those three young men, it was the hopeless situation of young people in the "fortress Cracow" (such was the official status of Cracow in those days of war), the insecurity of the future, the difficulty of earning one's living, the lack of contacts not only with foreign scientists, but even with the Polish ones — such was the atmosphere of this city in 1916. But all that

(1) A park surrounding the city.

did not prevent the three young men from spending a lot of time in cafés discussing mathematical problems amidst a noisy crowd. Banach did not mind the noise; for some reasons, known only to himself, he liked to sit quite near the orchestra.

I was studying the problem of the average convergence of Fourier series and this was the very question that I put to him in 1916 when I met him in the "Planty". For some time I had been trying to solve it myself. I was greatly surprised when, after a few days, Banach brought me a negative answer with a reservation which resulted from his ignorance of the Du Bois-Reymond example. Our common note was presented by S. Zaremba to the Academy of Sciences in 1917, to be published only in 1918. This delay was caused by the war.

Banach dreamt of being appointed mathematical assistant at Lwów Institute of Technology. This dream came true in 1920, when Antoni Łomnicki gave Banach the post. Since his arrival in Lwów, Banach's position was radically changed. His financial situation was secure. Banach married and settled down in the University building on St. Nicolas Str. In 1922, his doctor's thesis appeared in the 3rd volume of Fundamenta Mathematicae: "Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales".

That was Banach's seventh paper, but the first to be dedicated to the theory of linear operations. In the same year he was created assistant professor. With regard to him, universitary usages were not observed — he was granted his doctor's degree although he never completed his formal studies. At that time he was 30 years old. There was no lack of acknowledgments from other parts either. In 1924 Banach became correspondent member of the Polish Academy of Sciences, in 1930 he was awarded the Prize of the City of Lwów, and, in 1939, he was given the Prize of the Academy. It is difficult to understand nowadays why in that Academy there was no seat available for this son of the streets of Cracow. But the Lwów mathematicians understood at once that Banach would make Polish mathematics famous. Before his appearance there was, strictly speaking, no Lwów school, because Sierpiński, soon after the First World War, returned to Warsaw from which he had been driven away by that war, and Janiszewski died soon afterwards. In the interwar period, the Lwów school might be characterized, first of all, by the theory of operations, for its main achievements were in this field. Banach took up linear functionals, such as the integral. He showed that the concept of the integral may be widened so as to embrace all functions while retaining the properties postulated by Lebesgue; although this concept is ineffective, the proof of existence and the method of carrying it out (Fund. Math. 1923) show Banach's power. His main work is a book on linear operations. Published in 1932 as the first volume of Mathematical

Monographs (Warsaw, VII+254 pages) it is nowadays well known in the whole mathematical world under the title "Théorie des opérations linéaires". Its success is due to the fact that owing to the so-called "Banach spaces" it is possible to solve in a general way many problems which formerly called for special treatment and considerable ingenuity. There were other mathematicians, both great and small, who had attempted before Banach to build up a theory of operations. I remember what the outstanding Göttingen mathematician, Edmund Landau, said of the "Operazioni distributive" written by Pincherle; "Pincherle has written a book without having proved any theorem" — it was quite true. But they were also some more formidable competitors. Let us read what Norbert Wiener, the author of cybernetics, writes in his autobiography, published in 1956 under the title: "I am a mathematician". He says that Fréchet, who was the first to give the form of a linear functional in the L^2 space could not make up his mind as regards a system of postulates which would determine such a general space that L^2 would be only one of the numerous examples. Wiener takes the credit for it himself. He tells us how Fréchet, whose guest Wiener was in Strassburg (in 1920, on the occasion of the Mathematical Congress) showed him Banach's article in "a Polish mathematical journal". Fréchet was excited by the fact that Banach had given, a few months earlier than Wiener, a system of axioms of infinitely dimensional vectorial space identical with Wiener's system. "So" says Wiener, "the new theory was for some time called: The Banach-Wiener space theory". "But", says Wiener, "I wrote a few more papers on those problems, and gradually gave it up" — "at present those spaces are justly called after Banach's name alone..."(2). After this statement, Wiener dedicated a few pages of his autobiography to that conflict and explains why he left the battlefield; he thought that Banach's theory was a formalism, which could not show to its credit a sufficiently rich stock of original theorems hitherto unknown — now he admits that he was wrong, for after 34 years which have elapsed since the Strassburg Congress the Banach theory is still popular as a tool of analysis and "only now begins to develop its full effectiveness as a scientific method". Banach's fame reached the United States even before the appearance of "Opérations linéaires". As early as 1934, in the Bulletin of the American Mathematical Society (vol. 40, p. 13-16), J. D. Tamarkin wrote in his review of Banach's book: "It presents a noteworth climax of a long series of investigations started by Volterra, Fredholm, Hilbert, Hadamard, Fréchet and Frederick Riesz, continued effectively by Stefan Banach and his pupils". And then "The theory of linear operations is in itself an exciting domain, but its importance is enhanced

(2) This name was introduced by Fréchet.

by numerous and beautiful applications". One of Banach's most gifted pupils, Stanisław Ulam, writes thus in the obituary published in July, 1946, in the Bulletin of the American Mathematical Society (v. 52, No. 7, (1946), p. 600 - 633): "We have recently received the news that Banach died in Europe soon after the end of the War. The great interest aroused by his work is a well-known fact in our country. Indeed, in one of the fields of his activity, in the theory of infinitely dimensional linear spaces, the American school has developped and is still supplying very important results. It is an astounding coincidence of scientific intuition, which has concentrated the efforts of numerous Polish and American mathematicians in the same field..." And next; "Banach's work has set off for the first time in a general case the success of geometrical and algebraic approach to the problems of linear analysis, reaching far beyond the rather formal discoveries of Volterra, Hadamard and their successors. His results embraced more general spaces than the works of such mathematicians as Hilbert, E. Schmidt, von Neumann, F. Riesz and others. Many American mathematicians, particularly the younger ones, have taken up the idea of geometrical and algebraic study of linear functional spaces, and this work is still (1946) going on vigorously and bringing about important results".

I think that these opinions of outstanding scientists (one of whom has played an essential part in the computation of thermo-nuclear hydrogen reaction) may suffice as a proof that Banach knew how to take leading place in the development of an exceedingly important and new chapter of analysis, getting to the fore of a group of eminent mathematicians, who had already tried their forces in a similar trend.

May I be allowed to say for my part, as a witness of Banach's work, that he was possessed of a lucidity of thought which Kazimierz Bartel([3]) called once "even unpleasant...". He would never rely on a lucky stroke, he never expected that conjectures desirable at the time would prove true; he often said that "hope is the mother of fools". He adopted this contemptuous attitude to optimism not only in mathematics, but also towards political prophecies.

He was like Hilbert; he attacked the problems directly — after excluding through examples all side routes, he concentrated all his forces on the only way left, leading straight to the aim. He believed that logical analyzing of the problem, like a chessplayer analyzes a difficult position, must bring him to a proof or to the refutation of the theorem.

Banach's importance is not limited to what he had achieved himself in the theory of linear operations. On the list of his 58 works we find

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([3]) K. Bartel, Professor of the Lwów Institute of Technology, prime minister, murdered by the Gestapo in 1941.

studies written in collaboration with other mathematicians and books concerning other domains. The paper on the decomposition of sets into congruent parts, written together with Tarski (Fund. Math. 6 (1924), p. 244-277), may be placed in both those classes. It is a subject which reminds one of the school method of proving Pythagoras' theorem by cutting a large square into parts of which two small squares can be made; here, in three dimensions, the result is unexpected: a sphere can be cut into several parts of which two spheres can be made, each as big as the original one. Personally I was greatly impressed by a short paper in the Proceedings of the London Mathematical Society (vol. 21, p. 95-97). The problem consists in finding an orthogonal system complete in L^2 but incomplete in L. Banach chooses a function $f(t)$ which is (L) integrable $\int_0^1 f(t)dt = 1$, but such that $\int_0^1 f^2(t)dt = \infty$; he denotes by $\{\varphi_n(t)\}$ the sequence of all trigonometric functions $\{\cos nt, \sin nt\}$ and defines the numerical sequence $\{c_n\}$ by the relation $\int_0^1 f(t)\varphi_n(t)dt = c_n$; if we now define the sequence $\{\psi_n(t)\}$ by $\psi_n(t) = \varphi_n(t) - c_n$, we shall of course have $\int_0^1 f(t)\psi_n(t)dt = 0$ for all n. If we orthogonalize and normalize the sequence $\{\psi_n\}$, we shall obtain the required sequence $\{\gamma_n(t)\}$. The cleverness of the proof lies in the fact that the auxiliary sequence $\{\varphi_n(t)\}$ is deprived of the property which we demand from the sequence sought. Banach's works on the convergences of functionals are also known; they were originated by one of his colleagues, generalized by Banach and given their final shape by S. Saks (1927, Fund. Math. 9, p. 50-61). Banach took an interest in the problem of complanation, i. e. definition of the concept of the area of curved surfaces. His definition, gave rise to some investigations after the war (e. g. such conducted now by Prof. Kovanko in Lwów) — unfortunately no one knows how to reproduce the essential lemma indispensable to show the conformity of the Banach definition with the classical ones. We regret to say that many valuable results of Banach and his school's work were lost to the great detriment of Polish science as a result of carelessness on the part of the school's members and, first of all, of Banach himself. Another of his beautiful ideas is the replacing the classical definition of the oscillation of the function $y = f(x)$ by one more fitting to the epoch of Lebesgue, namely by the integral $\int_{-\infty}^{\infty} L(\eta)d\eta$, where $L(\eta)$ denotes the number of intersections of the curve $y = f(x)$ with the straight line $y = \eta$; the readers may find it interesting to know that this approach has a practical significance, e. g. it permits a quick calculation in "dollar-days" of the bank credits imprisoned in factory storehouses in the form of raw materials waiting for treatment, I do not want to say any more about the numerous and important items on the list of the works of the originator of the Lwów school and the founder of the journal "Studia Mathematica", which has played a considerable part in the development of that school and in the history of the linear operations theory.

Let us revert to Banach's person and his immediate influence on his environment. Banach was appointed full professor in 1927, but neither before nor after this fact was he a professor in the solemn sense of the word. His lectures were excellent; he never lost himself in particulars, he never covered the blackboard with numerous and complicated symbols. He did not care for verbal perfection; all humanistic polish was strange to him and, throughout his life he retained, in his speech and manners, some characteristics of a Cracow street urchin. He found it very difficult to formulate his thoughts in writing. He used to write his manuscripts on loose sheets torn out of a notebook; when it was necessary to alter any parts of the text, he would simply cut out the superfluous parts and stick underneath a piece of clean paper, on which he would write the new version. Had it not been for the aid of his friends and assistants, Banach's first studies would have never got to any printing office. He hardly ever wrote any letters and never answered questions addressed to him by post. He did not relish any logic research although he understood it perfectly. Neither was he attracted by any practical applications of mathematics, although he could certainly go in for them if he wanted to do so — a year after taking his doctor's degree he lectured on mechanics at the Institute of Technology. He used to say that mathematics is marked by specific beauty and can never be reduced to any rigid deductive system, since, sooner or later, it will burst any formal framework and create new principles. It was not the utilitarian but the specific value of mathematical theories that counted with him. His foreign competitors in the theory of linear operations either dealth with spaces that were too general, and that is why they either obtained only trivial results, or assumed too much about those spaces, which restricted the extent of the applications to few and artificial examples — Banach's genius reveals itself in finding the golden mean. This ability of hitting the mark proves that Banach was born a high class mathematician.

Banach could work at all times and everywhere. He was not used to comfort and he did not want any. A professor's earnings ought to have supplied amply all his needs. But his love of spending his life in cafés and a complete lack of bourgeois thrift and regularity in everyday affairs made him contract debts, and, finally, he found himself in a very difficult situation. In order to get out of it he started writing textbooks. Thus the "Differential and Integral Calculus" came to life in two volumes, the first of which was edited by the Ossolineum (1929, 294 pages) and the second by the Książnica-Atlas (1930, 248 pages). This manual, written in a concise and clear way, has enjoyed and is still enjoying a great popularity with university students in the first two years of their studies. The writing of secondary school textbooks for arithmetic, algebra and geometry took up a lot of Banach's time and effort. He wrote them in collaboration with Sierpiński and Stożek. Some of them were written by himself. He never copied any of the existing textbooks. Thanks to his coaching experience, Banach realized very well that every definition, every deduction and every exercise is a problem for the author of a schoolbook who cares for its didactic value. In my opinion Banach lacked only one of the many talents that an author of schoolbooks needs: spacial imagination. Banach's "Mechanics for Academic Schools" (Mathematical Monographs 8, 9) is the fruit of the experience collected during his numerous lectures on mechanics at the Institute of Technology. This two-book course, published in 1938, was issued once more in 1947, and, a few years ago, it was translated into English.

In order to give an account of Banach's importance for science in general, and for Polish science in particular, we should mention the names of his immediate followers: Mazur and Orlicz are his direct pupils; they represent the theory of operations in Poland. Their names to be seen today on the cover of "Studia Mathematica", signify the direct continuation of the Banach scientific program which found an expression in this journal. Stanisław Ulam (who owes his mathematical initiation to Kuratowski) after taking a doctor's degree also entered Banach's orbit. Banach with Mazur and Ulam formed the most important corner at the Scottish Café in Lwów. That was the place of gatherings to which Ulam alludes in the already quoted obituary: "it was hard to outlast or outdrink Banach during those sessions". There was even a session which lasted 17 hours — its result was the proof of a certain important theorem pertaining to the Banach spaces — but nobody took it down at the time and nobody can reproduce it today... Probably the top of the table covered with pencil marks was washed as usual by the Café servants. Such was the lot of a good many theorems proved by Banach and his followers. It was, therefore, a great merit on the part of Mrs. Łucja Banach — who is already lying at rest in the Wrocław cemetery — to buy a thick notebook with a stiff cover and to entrust it to the headwaiter of the Scottish Café. The problems were entered on the first pages of the successive leaves, so that the answers, if and when they were found, could be entered on the empty pages next to the text of the questions. This original "Scottish book" was at the disposal of every mathematician who asked for it in the Café. In some cases prizes were promised for solutions — they ranged from a small cup of black coffee to a live goose. Nowadays, he who smiles tolerantly on hearing about such ways of cultivating mathematics should try to understand that, according to Hilbert's opinion, the formulation of a problem is half the solution, and the list of unsolved and proclaimed problems makes people seek for a solution and is a challenge to all those who are not afraid to attempt difficult tasks. This state of mental emergency creates a scientific atmosphere. Among those of Banach's students who fell at the hands of murderers wearing uniforms adorned with the svastika, the most outstanding was undoubtedly P. J. Schauder, the laureate of the international Metaxas Prize, which was awarded to him and Léray *ex aequo*. It was Schauder who noticed how important were the Banach spaces for the boundary problems of partial differential equations. The difficulty lay in the selection of proper norms; it was overcome by Schauder and, thanks to that scientist, young at that time, the palm of victory in such a classical theory as partial differential equations was shared by France and Poland.

The later history of Banach's life passed under the shadow of the Second World War. In 1939-1941 he was dean of the Faculty in Lwów University, and even a correspondent member of the Kieff Academy, but after the German invasion (at the end of June 1941) he had to feed the lice in Professor Weigel's Bacteriological Institute. He spent several weeks in prison, because some people engaged in smuggling German marks were found in his home. By the time the case was cleared, Banach succeeded in proving a new theorem ([4]).

Banach was, first of all, a mathematician. He did not take much interest in politics, although he had a shrewd approach to every situation in which he happened to find himself. Nature did not impress him at all. Fine arts, literature, the theatre were to him second-rate amusements, which could, at their best, fill up the few short intervals in his work — on the other hand, he enjoyed a merry company and a drink. That is why the concentration of all his mental energy in one direction found no impediment. He did not like to delude himself and he knew very well that there is only a very small percentage of people who can understand mathematics. One day he said to me, "I'll tell you something, old chap! Humanities are more important in secondary schools than mathematics — mathematics is too sharp an instrument, it is not made for children to play with..."

It would be wrong to think that Banach was a dreamer, a selfdenying ascetic or apostle. He was a realist who even physically did not look like

([4]) Recently I have met in Chicago one of my former students. She told me that she had obtained her doctors degree from Banach; this proves his participation in the underground teaching in those days.

a candidate for a saint or even for a Tartuffe. I do not know whether there still exists, but there certainly did exist 30 years ago, the ideal of a Polish scientist, resulting not from the observation of reality but from the spiritual needs of the epoch which founds its expression in Stefan Żeromski's works. Such a scientist was supposed to work, far from all worldly pleasures, for a rather indefinite society, which forgave him a priori the ineffectiveness of his work, taking no heed of the fact that in other countries the greatness of scientists was gauged not by the greatness of personal self-denial, but by what they had done for science. In the interwar period the Polish intelligentsia was still under the suggestion of this self-mortification ideal, but Banach never submitted to it. He was healtly and strong, his realism was almost cynical, but he gave to Polish science, and particularly to Polish mathematics, more than anyone else. Nobody helped more than he to dispel the harmful opinion that, in scientific competition, lack of genius (or at least talent) can be compensated by other qualities, rather difficult to define. Banach was aware of his value; he realized fully what he was doing for science. He stressed his highland descent, and his attitude to sophisticated members of the intelligentsia without portfolio was a contemptuous one.

He lived to see the German collapse in Lwów, but a little later, on 31 August 1945 he died. He was buried at the cost of the Ukrainian Republic. One of the streets in Wrocław was named after him ([5]). His collective works will be published by the Polish Academy of Sciences.

His greatest merit is the overthrow and final annihilation of the Polish inferiority complex with regard to science, a complex camouflaged by the exaltation of mediocre thinkers. Banach never suffered from that complex — in his mind the spark of genius was combined with an astounding inward spirit, which told him again and again the words of Verlaine: "Il n'y a que la gloire ardente du métier!"([6]) — and mathematicians know very well that their craft consists in the same mystery as the craft of the poets.

([5]) Recently in Warsaw too.

([6]) "There is only one thing: the ardent glory of the craft!"