# GENERAL INDEX.


*Almost*, 16.
*Angle*, 310.
*Area*, of an interval, 59; Elementary, of a polyhedron, 165; of a continuous surface, 165.

*Boundary of a set*, 40.

*Centre*, of an open sphere, 40; of a closed sphere, 40.
*Chain of points*. 122.
*Classes of sets*, Additive, 7; Completely additive, 7; Additive in the weak sense, 7; Complete with respect to a measure 86.
*Closure of a set*, 40.
*Coefficients of an approximate differential*, 300.
*Complement of a set*, 6.
*Condition*, of Lipschitz for functions of two variables on an interval, 169; $(l_\alpha^+)$, 192; $(l_\alpha^-)$, 192; $(l_\alpha)$, 192; Lusin's, (N), 224; of Lipschitz for functions of a real variable on a set, 264; $(T_1)$, 277; $(T_2)$, 277; (S), 282; (D), 290; of Lipschitz for functions of two variables on a set, 304.
*Contingent*, 263.
*Coordinates of a point*, 56.
*Cubes*, 57; Open, 57; Half open, 57.
*Covering in the sense of Vitali*, 109.
*Curves*, 121; rectifiable on an interval, 122; Rectifiable, 122.

*Decomposition*, Jordan, of a function of a set, 11; Jordan, of a function of an interval, 62; Lebesgue, of a function of a set, 35; Lebesgue, of a function of an interval, 120.
*Densities*, Outer upper and lower, 128; Upper and lower, 129; Right-hand and left-hand, 215.
*Derivates*, General upper and lower, 106; (Ordinary) upper and lower, 106; Strong upper and lower, 106; Unique, 106; Extreme, 106; Bilateral, 108; Unilateral (right- and left-hand) 108; Dini, 108; relative to a set, 108; with respect to a function, 108; Intermediate, 108; Symmetrical, upper and lower, 149; with respect to a measure and a sequence of nets in a metrical space, 154; with respect to a sequence of nets of intervals, 188; Approximate right-hand and left-hand, upper and lower, 220; Approximate bilateral, upper and lower, 220; Opposite, 269.
*Derivative*, General, 106; Ordinary, 106; Strong, 106; Unilateral, 108; relative to a set, 108; with respect to a function, 109; with respect to a measure and a sequence of nets in a metrical space, 154; with respect to a sequence of nets of intervals, 188; of a complex function, 196; Roussel, 210; Approximate, 220.
*Diameter*, 40.
*Difference of sets*, 5.
*Differential*, Total, 300; Approximate, 300; Upper and lower, 309; Extreme, 309.
*Direction of a half-line*, 262.
*Distance*, of points, 39; of a point and a set, 40; of sets, 40; of points in an Euclidean space, 56; of points in the torus space, 157.

*Element of a set*, 4.
*Expressions of de Geöcze*, 171.

*Faces* of an interval, 57; of a polyhedron, 164.
*Families of sets*, see *Classes of sets*.
*Figure*, 58, Elementary, 58.

*Function of singularities*, of a function of a set, 35; of a function of an interval, 120.

*Functions (of a complex variable)*, Complex 195; Holomorphic. 195; Derivable, 196.

*Functions (of an interval)*, 59; Continuous, 59; continuous and discontinuous at a hyperplane, 60; Additive, 61; of bounded variation, 61; Monotone non-decreasing and non-increasing, 61; Absolutely continuous, 93; Singular, 93; Major and minor, 191; Major and minor, with respect to a function, 207. (See also *Functions (of a real variable)*.)

*Functions (of a point)*, Finite 6; Characteristic, of sets, 6; Simple, 7; Measurable 12; Integrable, 20; Upper and lower semi-continuous, 42; Continuous, 42; integrable in the Lebesgue-Stietjes sense, 65; integrable in the Lebesgue sense, 65; Summable, 65; Approximately continuous, 132; Equi-measurable, 143; Cylindrical, 159. (See also *Functions (of an interval)* and *Functions (of a real variable)*.)

*Functions (of a real variable)*, 96; of bounded variation, 96; Absolutely continuous, 96; Singular, 96; Regular, 97; Saltus-, 97; integrable in the sense of Newton, 186; $\mathcal{P}$-integrable, 201; integrable in the sense of Perron, 201; Approximately derivable, 220; of bounded variation (in the wide sense) 221; VB, 221; of generalized bounded variation (in the wide sense), 221; VBG, 221; absolutely continuous (in the wide sense), 223; AC, 223; generalized absolutely continuous (in the wide sense), 223; ACG, 223; of bounded variation in the restricted sense, 228; $VB_*$, 228; of generalized bounded variation in the restricted sense, 228; $VBG_*$, 228; absolutely continuous in the restricted sense, 231; $AC_*$, 231; generalized absolutely continuous in the restricted sense, 231; $ACG_*$, 231; $\mathcal{P}$-integrable, 241; $\mathcal{P}_*$-integrable, 241; continuous in the sense of Darboux, 272; Inner and outer, of a superposition, 286; Inverse, 286. (See also *Functions (of an interval)* and *Functions (of a point)*.)

*Functions (of a set)*, Additive, 8; Monotone, 8; Non-decreasing and non-increasing, 8; Absolutely continuous 30, 66; Singular, 30, 66.

*Functions (of two real variables)*, of bounded variation, 169; Absolutely continuous, 169; Totally differentiable, 300; Differentiable, 300; Approximately differentiable, 300.

*Graph*, of a function, 88; of a curve, 121.

*Half-plane*, 263, 264.

*Half-space*, 263, 306.

*Half-tangent*, Intermediate, 262.

*Hyperplane*, 57; orthogonal to an axis, 57; Intermediate tangent, 263; Extreme tangent, 263; Unique tangent, 263; Tangent, 263.

*Increment of a function*, 96.

*Integrals*, Definite, 19, 20, 46, 254; Indefinite, 29, 254; Lebesgue-Stieltjes, 65; Lebesgue, 65; Upper and lower Burkill, 166; Definite and indefinite Burkill, 166; Mean value, 178; Perron, 201, $\mathcal{P}$-, 201; Perron-Stieltjes, 207; $\mathcal{PS}$-, 207; Denjoy, in the wide sense, 241; Denjoy-Khintchine, 241; $\mathcal{D}$-, 241; Denjoy, in the restricted sense, 241; Denjoy-Perron, 241; $\mathcal{D}_*$-, 241; $\mathcal{L}$-, 241; $\mathcal{S}$-, 244; Compatible, 254.

*Interior of a set*, 40.

*Intervals*, 57; Closed, 57; Open, 57; Half-open, 57; Degenerate, 57; in the torus space, 157.

*Length*, Outer, of a set, 54; of a set 54; of an interval, 59; of arc of a curve, 122; of a curve, 122.

*Limit*, of a sequence of sets, 5; Upper and lower, of a sequence of sets, 5; of a sequence of points, 39; Approximate, upper and lower, 218; Approximate extreme, 219; Approximate, 219; Approximate bilateral and unilateral, 220.

*Lines*, Straight, 56, 57.

*Maximum*, of a function at a point, 42; Strict, 261.

*Measure*, 16; Outer Carathéodory, 43; of a set, 46; Outer, of a set, 46; Regular outer, 50; Outer Lebesgue, 65; Lebesgue, 65; - function, 117; Outer, in the torus space, 157; in the torus space, 158; Outer linear, of a set on a line, 297.

*Mesh of a net*, 153.

*Minimum*, of a function at a point, 42; Strict, 261.

*Neighbourhood of a point*, 40.
*Net*, of closed intervals, 57; of half open intervals, 57; in a metrical space, 153; Normal, of intervals, 188.
*Nucleus of a determining system*, 47.
*Number*, Characteristic, of a family of sets, 40.

*Operation* (A), 47.
*Ordinate-set of a function*, 88.
*Oscillation*, of a function of a point at a point, 42; of a function of an interval on a set, 60; of a function of an interval at a set, 60.

*Parameter*, of regularity of a set, 106; of a curve, 121.
*Part*, Common, of sets, 5; Non-negative and non-positive, of a function, 13; Absolutely continuous, of a function of an interval, 120; Real and imaginary, of a complex function, 195.
*Plane*, 56, 57; Whole, 264. (See also *Hyperplane*.)
*Points*, of a space, 6; of accumulation, 40; Isolated, 40; Internal, 40; of a curve, 121; of outer density, 128; of dispersion, 128; of density, 129; Right- and left-hand, of accumulation, 215; isolated on the right or the left, 215; Unilateral (right- and left-hand), of outer density, 215; of linear density in the direction of an axis, 298.
*Polyhedron*, 164.
*Portion of a set*, 41.
*Primitive of Newton*, 186.
*Product of sets*, 5; Combinatory, 82; Cartesian, 82.

*Radius*, of a neighbourhood, 40; of an open sphere, 40; of a closed sphere, 40.
*Relations*, Denjoy, 269.

*Sequences*, Convergent, of sets, 5; Ascending and descending, of sets, 5; Non-decreasing and non-increasing, of sets, 5; Monotone, of sets, 5; Convergent, of points, 40; Regular, of nets of intervals, 57; Regular, of nets in a metrical space, 153; of mean value integrals, 178; Normal, of nets of intervals, 188; Binary, of nets of intervals, 191.
*Sets*, Empty, 4; Enumerable, 4; Measurable, 7; Bounded, 40; Derived, 40; Closed, 40; Isolated, 40; Perfect, 40; Open, 40; Non-overlapping, 40; ($\mathfrak{F}$), 40; ($\mathfrak{G}$), 40; ($\mathfrak{B}$), 41; measurable ($\mathfrak{B}$), 41; Borel, 41; closed in a set, 41; open in a set, 41; everywhere dense in a set, 41; non-dense in a set, 41; of the first and the second category (in a set), 41; Separable, 41; measurable with respect to an outer Carathéodory measure, 44; regular with respect to an outer Carathéodory measure, 50; of zero length, 54; of zero area, 54; of zero volume, 54; of finite length, 54; of finite area, 54; of finite volume, 54; Linear, 56; Plane, 56; Ordinate-, 88; Measurable, in the torus space, 158; Cylindrical, in the torus space, 159.
*Side* of a hyperplane, 263; Empty, 263.
*Space*, Abstract, 6; Metrical, 39; Complete, 54; Euclidean, 56; Torus, 157; Whole, 306.
*Sphere*, Open, 40; Closed, 40.
*Square*, 57.
*Subdivision of a figure*, 165.
*Subsets*, 4.
*Sum of sets*, 5.
*Superposition of functions*, 286.
*Surfaces*, Continuous, 164.
*System*, Determining, 47; Degenerate determining, 49.

*Tangent*, Intermediate, 263; Extreme, 263; Unique, 263.
*Torus space*, 157.
*Translation*, of a set by a vector, 91; in the torus space, 158.

*Variation*, Relative (upper and lower) and absolute, of a function of a set, 10; Relative (upper and lower) and absolute, of a function of an interval, 61, 62; of a function of a real variable, 96; of a function of an interval at a set, 166; Weak, 221; Strong, 228.
*Vertex*, of a polyhedron, 164; of an angle, 310.
*Volume*, of an interval, 59; of an interval in the torus space, 157.